CORE MATHS TRANSITION WORK

This booklet contains work needed for the first term in year 12.

The exercises here will help you prepare the topics you will use and analyse once you start the course.

You need to make sure you are secure in your understanding of all these topics. Answers are given in a section at the end for you to check your work.

What is $4 + 3 \times 5$? It is not sensible to have two possible answers.
It has been agreed that calculations are done obeying certain rules:

First   Brackets and Division line
Second   Divide and Multiply
Third   Addition and Subtraction

EXAMPLES

1 $4 + 3 \times 5$ $= 4 + 15$ $= 19$

2 $10 \div 2 + 3$ $= 5 + 3$ $= 8$

3 $10 \div (2 + 3)$ $= 10 \div 5$ $= 2$

4 $(5 + 6) \times 3 + 4$ $= 11 \times 3 + 4$ $= 33 + 4$ $= 37$

5 $\dfrac{12}{11 - 8} - 3$ $= \dfrac{12}{3} - 3$ $= 4 - 3$ $= 1$

This is the same as $12 \div (11 - 8) - 3$.

Exercise 1.7

Do not use a calculator for this exercise.

1 Work these out.
(a) $7 + 6 \times 5$ (b) $7 - (6 - 2)$ (c) $24 \div 6 + 5$
(d) $7 \times 6 + 8 \times 2$ (e) $10 \div 5 + 8 \div 2$ (f) $(5 - 2) \times 7 + 9$
(g) $60 \div (5 + 7)$ (h) $60 \div 5 + 7$ (i) $4 \times 3 + 2$
(j) $4 \times (3 + 2)$ (k) $12 \times (20 - 2) \div 9$ (l) $36 \div (5 + 4)$
(m) $4 \times 12 \div 8 - 6$ (n) $9 \times 9 - 5 \times 5$ (o) $(9 + 5) \times (9 - 5)$
(p) $\dfrac{22 - 4}{17 - 8} + 12 \div 3$ (q) $\dfrac{6 \times 3 - 2}{2 \times 2} + 3 \times 8$ (r) $\dfrac{(3 + 7) \times 10 - 19}{(2 + 1) \times (8 - 5)} - 3 \times 3$

2 Choose from the four signs $+$, $-$, $\times$ and $\div$ to make these sums correct.
(a) $5 \quad 6 \quad 7 = 37$ (b) $5 \quad 6 \quad 7 = 47$ (c) $15 \quad 8 \quad 9 = 87$
(d) $15 \quad 8 \quad 9 = 129$ (e) $15 \quad 8 \quad 9 = 111$ (f) $15 \quad 5 \quad 3 = 6$
(g) $5 \quad 24 \quad 6 = 1$ (h) $19 \quad 19 \quad 7 \quad 0 = 1$ (i) $4 \quad 4 \quad 7 \quad 2 = 30$

3 Using all the numbers 6, 3, 2 and 1 in this order, brackets and the signs $+$, $-$, $\times$ and $\div$ make all the numbers from 1 to 10.
$$6 - 3 \times 2 + 1 = 1, \qquad 6 - 3 - 2 + 1 = 2, \qquad \text{and so on.}$$

4 The caretaker set out 17 rows of chairs. There are 15 chairs in each row. How many more chairs are needed to provide seats for 280 people?

5 Claire is 16 cm taller than Rachel. Their heights add up to 312 cm. How tall is Rachel?

6 The admission charges to a zoo are £4 for a child and £7 for an adult. Zoe is organising a trip to the zoo for a group of people and worked out that the total cost would be £336. She collected £84 from the adults in the group.
(a) How many children are in the group?
(b) What is the total number of people in the group?

Using formulae

The formula for the perimeter of a rectangle is $P = 2L + 2W$.
By substituting values for the length, L, and the width, W, you can calculate the value of P.

$P = 2L + 2W$
When $L = 3$ and $W = 5$,
$P = 2 \times 3 + 2 \times 5$
$= 6 + 10$
$= 16$

EXAMPLES

1 A joiner earns £W for working H hours. Her boss uses the formula
$W = 5H + 35$ to calculate her wage. Find her wage if she works for 40 hours.

$W = 5 \times 40 + 35$
$= 200 + 35$
$= £235$

2 $H = 3(4x - y)$. Find the value of H when $x = 5$ and $y = 7$.

$H = 3(4x - y)$
$= 3(4 \times 5 - 7)$
$= 3(20 - 7)$
$= 3(13)$
$= 39$

3 $A = pq - r$
What is the value of A when (a) $p = 2$, $q = -2$ and $r = 3$,
(b) $p = 3$, $q = 2$ and $r = -2$?

(a) $A = 2 \times (-2) - 3$
$= -4 - 3$
$= -7$

(b) $A = 3 \times 2 - (-2)$
$= 6 + 2$
$= 8$

Exercise 14.2   Do not use a calculator for questions 1 to 15.

1 Find the value of $x + 3$ when
(a) $x = 2$ (b) $x = 7$ (c) $x = -2$

2 Find the value of $4a$ when
(a) $a = 5$ (b) $a = 7$ (c) $a = -3$

3 Find the value of $p - 3$ when
(a) $p = 4$ (b) $p = 5$ (c) $p = -3$

4 Find the value of $5 - d$ when
(a) $d = 3$ (b) $d = 7$ (c) $d = -3$

5 Find the value of $2x + 3$ when
(a) $x = 5$ (b) $x = -5$

6 Find the value of $4 - 2y$ when
(a) $y = 3$ (b) $y = -3$

7 Find the value of $4x$ when
(a) $x = \frac{1}{2}$ (b) $x = \frac{1}{4}$ (c) $x = \frac{1}{8}$

8 Find the value of $5y$ when
(a) $y = 0.2$ (b) $y = 0.3$ (c) $y = 0.5$

9 Find the value of $2x + 5$ when
(a) $x = \frac{1}{2}$ (b) $x = 0.4$

10 Find the value of $4y - 1$ when
(a) $y = \frac{1}{4}$ (b) $y = 0.5$

11 Find the value of $5 - 3g$ when
(a) $g = 0.5$ (b) $g = \frac{1}{3}$

12 $F = 5(v + 6)$.
What is the value of F when $v = 9$?

13 $V = 2(7 + 2x)$.
What is the value of V when $x = 3$?

14 $P = 3(5 - 2d)$.
What is the value of P when $d = 4$?

15 $C = 8(p + q)$.
What is the value of C when
(a) $p = 5$ and $q = 8$,
(b) $p = 6$ and $q = -2$?

16 $S = ax + 4$.
    What is the value of S when
    (a) $a = 12$ and $x = 3$,
    (b) $a = 3$ and $x = -2$,
    (c) $a = 5$ and $x = 0.4$?

17 $T = a(x + 4)$.
    What is the value of T when
    (a) $a = 5$ and $x = 3$,
    (b) $a = 2$ and $x = -5$,
    (c) $a = -3$ and $x = 2$,
    (d) $a = -3$ and $x = -6$?

18 $K = ab + c$.
    Work out the value of K when
    (a) $a = 3$, $b = -2$ and $c = 5$,
    (b) $a = 5$, $b = 3$ and $c = -2$.

19 $L = xy - z$.
    Work out the value of L when
    (a) $x = 2$, $y = 3$ and $z = -4$,
    (b) $x = -4$, $y = 2$ and $z = 3$.

20 The number of matches, M, needed
    to make a pattern of P pentagons is
    given by the formula $M = 4P + 1$.
    Find the number of matches needed
    to make 8 pentagons.

21 The distance, d metres, travelled by a
    lawn mower in t minutes is given by
    the formula: $d = 24t$.
    Find the distance travelled by the
    lawn mower in:
    (a) 4 minutes,   (b) 30 minutes,
    (c) 90 seconds.

22 Convert these temperatures from Fahrenheit
    to Centigrade using the formula:
    $C = (F - 32) \div 1.8$
    (a) 14°F   (b) −4°F
    (c) −22°F   (d) −40°F

23 Convert these temperatures from Centigrade
    to Fahrenheit using the formula:
    $F = C \times 1.8 + 32$
    (a) 10°C   (b) −10°C
    (c) −30°C   (d) −40°C

24 $T = 45W + 30$ is used to calculate the time
    in minutes needed to cook a joint of beef
    weighing W kilograms.
    How many minutes are needed to cook a
    joint weighing 2.4 kg?

25 The voltage, V volts, in a circuit with
    resistance, R ohms, and current, I amps, is
    given by the formula $V = IR$.
    Find the voltage in a circuit when
    $I = 12$ and $R = 20$.

26 A simple formula for the motion of a car is
    $F = ma + R$.
    Find F when $m = 500$, $a = 0.2$ and $R = 4000$.

27 The cost, £C, of n units of gas is calculated
    using the formula $C = 0.08n + 3.5$.
    Calculate the cost of 458 units of gas.

28 The formula $v = u + at$ gives the speed v of
    a particle, t seconds after it starts with
    speed u. Calculate v when $u = 7.8$,
    $a = -10$ and $t = \frac{3}{4}$.

Substitution into formulae with powers and roots

Exercise 14.3

Do not use a calculator for questions 1 to 12.

1 $S = a^2$. Find the value of S when
   (a) $a = 3$ (b) $a = -3$

2 $S = 2a^2$. Find the value of S when
   (a) $a = 3$ (b) $a = -3$

3 $S = (2a)^2$. Find the value of S when
   (a) $a = 3$ (b) $a = -3$

4 $S = \frac{1}{2} p^2$. Find the value of S when
   (a) $p = 8$ (b) $p = -8$

5 $S = \left(\frac{1}{2} p\right)^2$. Find the value of S when
   (a) $p = 8$ (b) $p = -8$

6 What is the value of $3a^2 - 9$ when
   (a) $a = 4$,   (b) $a = 5$,
   (c) $a = -4$,   (d) $a = -5$?

7 What is the value of x^3 when
   (a) $x = 3$,   (b) $x = -3$,
   (c) $x = 5$,   (d) $x = -5$?

8 What is the value of $2t^3$ when
   (a) $t = 4$,   (b) $t = -4$?

What is the value of $3x - x^3$ when

Work out the value of T when
(a) $a = 9$ and $b = 16$,
(b) $a = 3$ and $b = \frac{4}{3}$.

Work out the value of S when
(a) $p = 2$ and $q = 32$,
(b) $p = 2\frac{1}{4}$ and $q = 1\frac{1}{2}$,
(c) $p = 54$ and $q = -0.5$.

Work out the value of L when
(a) $m = 6$ and $n = 8$,
(b) $m = 0.3$ and $n = 0.4$.

Calculate $5\sqrt{x}$ when
(a) $x = 27$,
(b) $x = 0.4$.
Give your answers correct to one decimal place.

Calculate $\sqrt{5x}$ when
(a) $x = 27$,
(b) $x = 0.4$.
Give your answers correct to two significant figures.

15. The formula $F = \dfrac{mv^2}{r}$ describes the motion of a cyclist rounding a corner. Find F when $m = 80$, $v = 6$ and $r = 20$.

16. The height, h metres, of a bullet after t seconds, is given by the formula $h = ut - \frac{1}{2}gt^2$, where $u\,\text{ms}^{-1}$ is the initial vertical speed and $g\,\text{ms}^{-2}$ is the acceleration due to gravity.
Find h when $u = 200$ and $t = 1\frac{3}{5}$.
Take $g = 9.8$.
Give your answer to a suitable degree of accuracy.

17. The time T, for a pendulum to make a complete swing is given by the formula:
$$T = 2\pi\sqrt{\frac{l}{g}}$$
(a) Calculate the value of T when $l = 0.8$ and $g = 9.8$.
(b) Calculate the value of T when $l = 1\frac{1}{2}$ and $g = 9.8$.
Take π to be 3.14 or use the π key on your calculator.

18. Use the formula $v = \sqrt{u^2 + 2as}$ to calculate the value of v when
(a) $u = 2.4$, $a = 3.2$, $s = 5.25$,
(b) $u = 9.1$, $a = -4.7$, $s = 3.04$.
Give your answers correct to one decimal place.

Rearranging formulae

Sometimes it is easier to use a formula if you **rearrange** it first.

EXAMPLES

1 $k = \dfrac{8m}{5}$

Rearrange the formula to give m in terms of k.

$k = \dfrac{8m}{5}$

Multiply both sides by 5.
$5k = 8m$
Divide both sides by 8.
$\dfrac{5k}{8} = m$

We say we have **rearranged the formula**
$k = \dfrac{8m}{5}$ to make m the **subject** of the formula.

2 $y = 2x + 8$

Make x the subject of the formula.

$y = 2x + 8$

Subtract 8 from both sides.
$y - 8 = 2x$
Divide both sides by 2.
$\frac{1}{2}y - 4 = x$
y is the subject of $y = 2x + 8$,
x is the subject of $x = \frac{1}{2}y - 4$.

155

10 Which rectangle has the greater percentage shaded?

A

B

11 In an ice hockey competition Team A won 8 out of the 11 games they played whilst Team B won 5 of their 7 games.

Which team has the better record in the competition?

Expressing one quantity as a percentage of another

To work out one number as a percentage of another there are two steps.

Step 1	Write the numbers as a fraction.
Step 2	Change the fraction to a percentage.

EXAMPLES

○ ○ ○ ○ ○
The numbers in the fraction must be in the **same** units.

1 What is 30p as a percentage of £2?

£2 = 200p
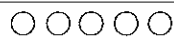
Step 1
30p as a fraction of 200p is $\frac{30}{200}$.

Step 2
$\frac{30}{200} \times 100 = 30 \times 100 \div 200 = 15\%$

So 30p as a percentage of £2 is 15%.

2 A newspaper contains 48 pages, 6 of which are Sports pages.
What percentage of the pages are Sports pages?

Step 1
6 out of 48 pages are Sports pages.
$\frac{6}{48} = 6 \div 48 = 0.125$

Step 2
$0.125 \times 100 = 12.5$
12.5% of the pages are Sports pages.

Exercise 8.2   Do not use a calculator for questions 1 to 6.

1 What is
(a) 30 as a percentage of 50,
(b) 4 as a percentage of 25,
(c) 7 as a percentage of 10,
(d) 42 as a percentage of 200,
(e) 63 as a percentage of 300?

2 What is
(a) 64 pence as a percentage of £2,
(b) 15 km as a percentage of 120 km,
(c) 30 cm as a percentage of 600 mm,
(d) £3600 as a percentage of £4000,
(e) 18 pence as a percentage of £0.60?

3 There are 8 yellow fruit drops in a packet of 25 fruit drops.
What percentage of the fruit drops are yellow?

4 A school has 800 pupils of which 160 are in Year 11.
What percentage of pupils are in Year 11?

5 James saved £30 and then spent £9.
What percentage of his savings did he spend?

6 A Youth Club has 200 members.
80 of the members are boys.
(a) What percentage of the members are boys?
(b) What percentage of the members are girls?

7 240 people took part in a survey.
30 of them were younger than 18.
What percentage were younger than 18?

8 A bar of chocolate has 32 squares.
Jane eats 12 of the squares.
What percentage of the bar does she eat?

9 Billy earns £4.50 per hour.
He gets a wage rise of 27 pence per hour.
What is his percentage wage rise?

10 What is
(a) £2 as a percentage of £6,
(b) 80 km as a percentage of 120 km,
(c) 20 cm as a percentage of 180 cm,
(d) £1530 as a percentage of £3600,
(e) £105.09 as a percentage of £186?

11 A new car costs £13 500.
The dealer gives a discount of £1282.50.
What is the percentage discount?

12 There are 600 pupils in Years 9 to 13 of
a High school.
360 pupils are in Years 10 and 11.
15% of pupils are in Years 12 and 13.
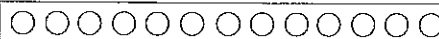What percentage of pupils are in Year 9?

Finding a percentage of a quantity

EXAMPLE

Find 20% of £56.

Step 1   Divide by 100.
£56 ÷ 100 = £0.56

Step 2   Multiply by 20.
£0.56 × 20 = £11.20
So 20% of £56 is £11.20.

Steps 1 and 2 could be done in the opposite order.
£56 × 20 = £1120
£1120 ÷ 100 = £11.20

To find 1% of a quantity divide
the quantity by 100.

To find 20% of a quantity multiply
1% of the quantity by 20.

This is the same as the method
you would use to find $\frac{20}{100}$ of a
quantity.

Percentage change

EXAMPLES

1 A shirt normally priced at £24 is
reduced by 15% in a sale.
How much does it cost in the sale?

Reduction in price = 15% of £24
15 ÷ 100 × 24 = 0.15 × 24 = 3.6
15% of £24 = £3.60
The shirt costs £24 − £3.60 = £20.40.

2 There are 440 g in a normal packet of Rice
Crunchies. A special offer packet contains
30% more than the normal packet.
How many grams of Rice Crunchies are
there in the special offer packet?

Extra contents = 30% of 440 g
= 440 ÷ 100 × 30
= 132 g
440 + 132 = 572
There are 572 g in a special offer packet.

Do not use a calculator in this exercise.

1 Find
(a) 20% of £80
(b) 75% of £20
(c) 30% of £220
(d) 15% of £350
(e) 5% of £500
(f) 20% of £150
(g) 9% of £300
(h) 20% of 20 m
(i) 30% of 80 kg
(j) 35% of 800
(k) 45% of £25
(l) 60% of 20

2 Garry has 300 marbles.
20% of the marbles are blue.
35% of the marbles are red.
The rest of the marbles are white.
(a) How many marbles are blue?
(b) How many marbles are red?
(c) What percentage of the marbles are white?

3 Tim invests £400 in a building society.
He earns 5% interest per year.
How much interest does he get in one year?

4 There are 450 seats in a theatre.
60% of the seats are in the stalls.
How many seats are in the stalls?

5 A salesman earns a bonus of 3% of his weekly sales.
How much bonus does the salesman earn in a week when his sales are:
(a) £1400, (b) £2350?

6 Jenny gets a 15% discount on a theatre ticket.
The normal cost is £13.
How much does she save?

7 Dipak earns £150 per week.
He gets a wage rise of 3%.
How much extra does he earn each week?

8 In a school of 1200 pupils 45% are boys.
(a) How many are girls?

30% of the girls are under 13.
(b) How many girls are under 13?

9 A dozen biscuits weigh 720 g.
The amount of flour in a biscuit is 40% of the weight of a biscuit.
What is the weight of flour in **each** biscuit?

10 Increase:
(a) £400 by 20%   (b) £300 by 40%
(c) £2000 by 40%   (d) £600 by 80%
(e) £3000 by 15%   (f) £900 by 40%
(g) £50 by 60%   (h) £10 by 30%
(i) £15 by 10%   (j) £50 by 15%

11 Decrease:
(a) £600 by 30%   (b) £800 by 25%
(c) £2500 by 20%   (d) £250 by 40%
(e) £12 000 by 15%   (f) £7000 by 35%
(g) £600 by 15%   (h) £55 by 90%
(i) £42 by 20%   (j) £63 by 35%

12 A mobile telephone company offers a 20% discount on calls made in March.
The normal cost of a peak time call is 30 pence per minute.
How much does a peak time call cost in March?

13 Abdul earns £200 per week.
He gets a wage rise of 7.5%.
What is his new weekly wage?

14 Prices in a sale are reduced by 18%.
The normal price of a shirt is £22.50.
Calculate its sale price.

15 A packet of breakfast cereal contains 660 g.
A special offer packet contains an extra 15%.
How many grams of breakfast cereal are in the special offer packet?

16 The price of a car is £12 500.
A dealer gives a 7% discount.
What is the price of the car after the discount?

17 35% of a magazine is pictures.
In the magazine there are 60 pages.
Each page is 25 cm long and 16 cm wide.
What is the area of pictures in the magazine?

More complicated percentage problems

Problems involving percentages can involve more complicated calculations.

EXAMPLES

1 Calculate 4×10^6 as a percentage of 2×10^8.

$$\frac{4 \times 10^6}{2 \times 10^8} = 0.02$$

$$0.02 \times 100 = 2\%$$

2 Calculate 30% of 8×10^4.

$$30 \div 100 \times 8 \times 10^4$$
$$= 24\,000$$

3 1.2×10^{10} steel cans were used in 1992. 1.4×10^9 of these were recycled. Calculate the percentage of steel cans that were **not** recycled in 1992.

$$\frac{1.4 \times 10^9}{1.2 \times 10^{10}} = 0.1166...$$

$$0.1166... \times 100 = 11.66...$$
$$= 11.7 \text{ to 3 significant figures.}$$
Percentage recycled $= 11.7\%$
$100 - 11.7 = 88.3$
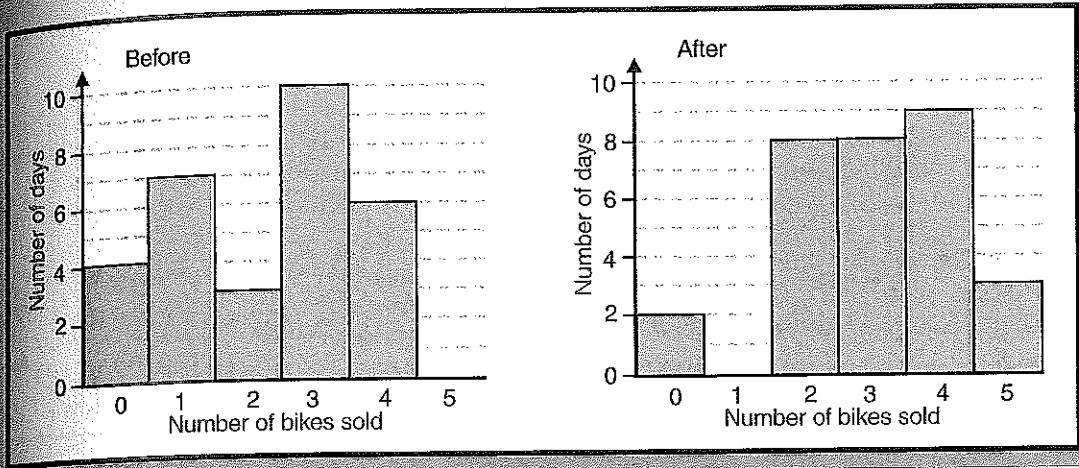Percentage not recycled $= 88.3\%$

Exercise 8.4

Use a calculator in this exercise.
Where appropriate give your answers to
3 significant figures.

1 Prices in a sale are reduced by 24%.
The normal price of a shirt is £25.50.
Calculate its sale price.

2 (a) Jane's salary of £14 050 is increased by 3.5%. Calculate her new salary.
(b) Petrol costs 81.9 pence a litre. What does it cost after a 2.4% increase?
(c) Milk costs 28 pence a pint. How much does it cost after a 12.5% increase?

3 (a) The price of a gold watch is £278. What does it cost with a 12% discount?
(b) The price of a used car is £5200. What does it cost with a 9.5% discount?
(c) The price of a new kitchen is £3650. What does it cost with a 35% discount?

4 A 5 litre can of paint covers an area of 28 m².
Harry buys 3 cans of paint to cover 70 m².
What percentage of the paint does he use?

5 A car was valued at £13 500 when new. After one year it lost 22% of its value. At the end of two years it was sold for £8200.
(a) What was the value of the car after one year?
(b) What percentage of its original value did the car lose in its second year?

6 In 1999 house prices increased by 9.6%.
In 2000 house prices increased by 7.4%.
A house was valued at £78 000 at the beginning of 1999. What was the value of the house at the end of 2000?

7 Ben invests £650 in a building society. He earns 5.25% interest in the first year.
(a) How much interest does he earn?

Ben leaves his original £650 plus the interest he has earned in the building society.
He earns 6.05% in the second year.
(b) How much interest does Ben earn in the second year?

The graphs show the monthly sales of bicycles before and after a marketing campaign. Calculate the medians and the ranges. Use your results to compare 'Before' with 'After'.

Before

Number of days (y-axis: 0, 2, 4, 6, 8, 10)

Number of bikes sold (x-axis: 0, 1, 2, 3, 4, 5)

After

Number of days (y-axis: 0, 2, 4, 6, 8, 10)

Number of bikes sold (x-axis: 0, 1, 2, 3, 4, 5)

Which is the best average to use?

Many questions in mathematics have definite answers. This one does not.
Sometimes the mean is best, sometimes the median and sometimes the mode. It all depends on the situation and what you want to use the average for.

EXAMPLES

① A youth club leader gets a discount on cans of drinks if she buys all one size.
She took a vote on which size people wanted.
The results were as follows:

Size of can (ml)	100	200	330	500
Number of votes	9	12	19	1

Mode = 330 ml
Median = 200 ml
Mean = 245.6 ml, correct to one decimal place.

Which size should she buy?

The mean is no use at all because she can't buy cans of size 245.6 ml.
Even if the answer is rounded to the nearest whole number (246 ml), it's still no use.
The median is possible because there is an actual 200 ml can.
However, only 12 out of 41 people want this size.
In this case the **mode** is the best average to use, as it is the most popular size.

② A lecturer sets a unit test. He wants to choose a minimum mark for a distinction so that 50% of his students get this result. Should he use the modal mark, the median mark or the mean mark?

The median mark is the middle mark, so half the students will get the median mark or higher.
The lecturer must use the median mark.
Explain why the modal mark and the mean mark are not suitable.
The median mark cannot be decided until after the test.
Explain why.

In questions 1 to 3 find all the averages possible. State which is the most sensible and why.

(1) On a bus: 23 people are wearing trainers,
                  10 people are wearing boots,
                    8 people are wearing lace-up shoes.

(2) 20 people complete a simple jigsaw. Their times, in seconds, are recorded.
5, 6, 8, 8, 9, 10, 11, 11, 12, 12, 12, 15, 15, 15, 15, 18, 19, 20, 22, 200.

(3) Here are the marks obtained by a group of 11 students in a mock exam. The exam was marked out of 100.
5, 6, 81, 81, 82, 83, 84, 85, 86, 87, 88.

(4) The times for two swimmers to complete each of ten 25 m lengths are shown below.

Swimmer A	30.1	30.1	30.1	30.6	30.7	31.1	31.1	31.5	31.7	31.8	
Swimmer B	29.6	29.7	29.7	29.9	30.0	30.0	30.1	30.1	30.1	44.6	

Which is the better swimmer? Explain why.

(5) The table shows the number of runs scored by two batsmen in several innings.

Batsman A	0	0	10	12	20	22	50	51	81	104		
Batsman B	0	24	25	27	28	30	33	34	44	45	46	96

Which is the better batsman? Explain why.

What you need to know

- There are three types of **average**: the **mode**, the **median** and the **mean**.
  The **mode** is the most common amount.
  The **median** is the middle amount (or the mean of the two middle amounts) when the amounts are arranged in order of size.
  $$\textbf{Mean} = \frac{\text{Total of all amounts}}{\text{Number of amounts}}$$

- The **range** is a measure of **spread**.
  Range = highest amount − lowest amount

- To find the mean of a **frequency distribution** use:
  $$\text{Mean} = \frac{\text{Total of all amounts}}{\text{Number of amounts}} = \frac{\Sigma fx}{\Sigma f}$$

- To find the mean of a **grouped frequency distribution**, first find the value of the midpoint of each class. Then use:
  $$\text{Estimated mean} = \frac{\text{Total of all amounts}}{\text{Number of amounts}} = \frac{\Sigma fx}{\Sigma f}$$

- Choosing the best average to use:
  When the most **popular** value is wanted use the **mode**.
  When **half** of the values have to be above the average use the **median**.
  When a **typical** value is wanted use either the **mode** or the **median**.
  When all the **actual** values have to be taken into account use the **mean**.
  When the average should not be distorted by a few very small or very large values do **not** use the mean.

1 The lateness of 12 buses is recorded.
The results, in minutes, are shown.

5   6   7   8   8   10   10   10   11   12   13   14

(a) (i) What is the range of lateness for these buses?
    (ii) Calculate the mean lateness for these buses.

The lateness of 12 trains is also recorded.
The range in lateness for these trains is 14 minutes and the mean lateness is 5 minutes.

(b) Compare and comment on the lateness for these buses and trains.

2 (a) Find the range and mode of these prices.
    (b) Calculate the median and mean price of a bottle of milk.

Price of a bottle of milk in different shops
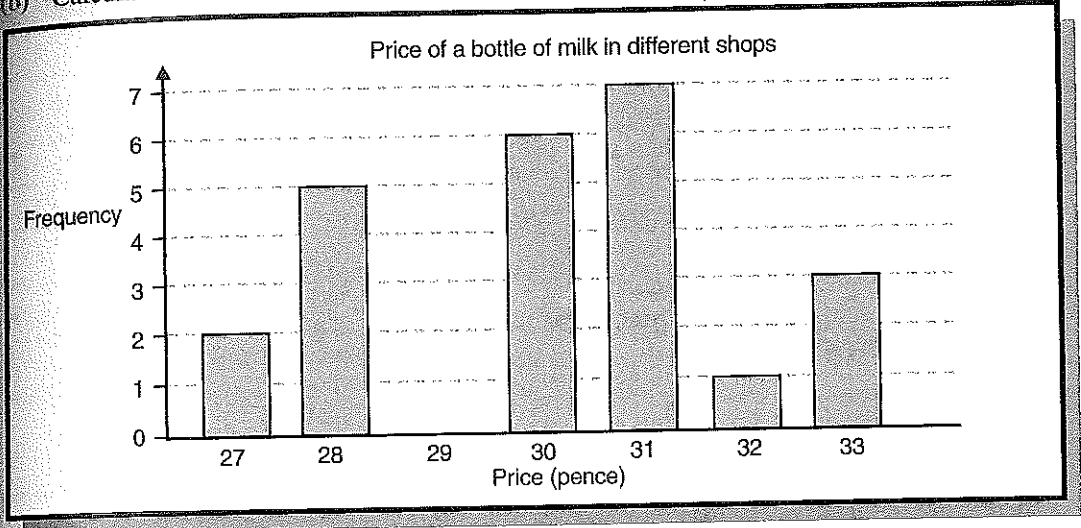
3 David is playing cricket.
The table shows the number of runs
he has scored off each ball so far.

Number of runs	0	1	2	3	4	5	6
Number of balls	3	8	4	3	5	0	2

(a) (i) What is the median number of runs per ball?
    (ii) Calculate the mean number of runs per ball.
Off the next five balls, David scores the following runs: 4, 4, 5, 3 and 6.
(b) (i) Calculate the new median.
    (ii) Calculate the new mean.
(c) Give a reason why the mean is used, rather than the median, to give the average
    number of runs scored per ball.   AQA

4 Sita plays in the school netball team.
After 8 games her mean score is 6.5 goals.
(a) What is the total number of goals that Sita has scored
    in these 8 games?
(b) Selection for the County team is made after 9 games
    have been played.
    Sita will be chosen for the County team if her mean
    score is 7 goals or more.
    What is the smallest number of goals she must score
    in the ninth game in order to be chosen?
(c) In the ninth game she actually scores 10 goals.
    Does her mean score of goals increase or decrease?
    Explain your answer.   AQA

Box plots

Box plots (or **box and whisker diagrams**) provide a useful way of representing the range, the median and the quartiles of a set of data.
They are also useful for comparing two (or more) distributions.

The graph shows the cumulative frequency distribution of the masses of 80 fish.
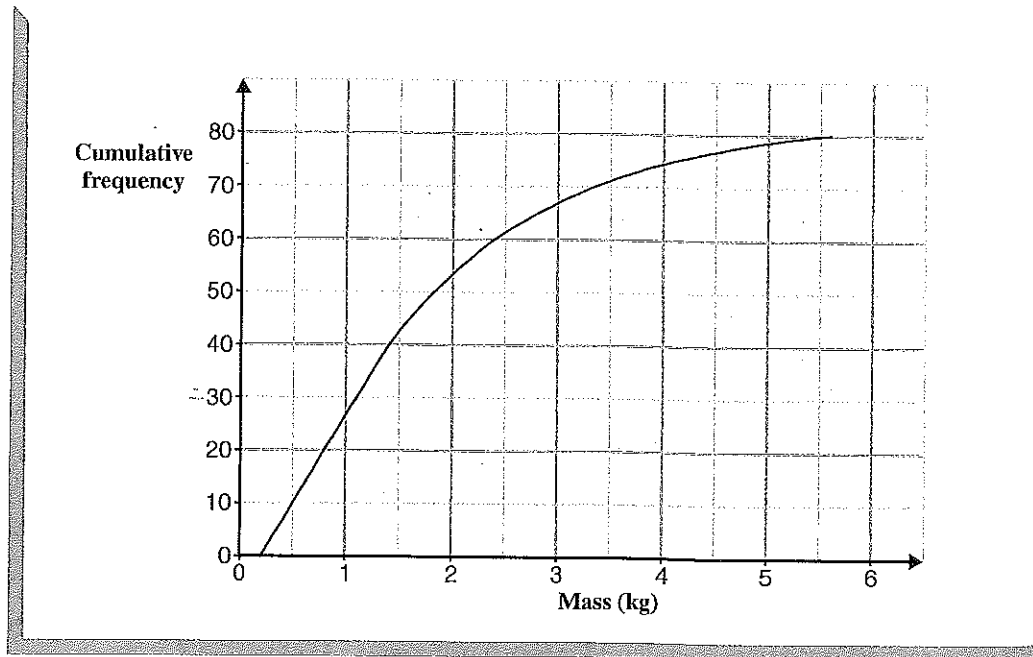
From the graph we can read off the following information:

- the minimum mass is 0.2 kg,
- the maximum mass is 5.6 kg,
- the median mass is 1.4 kg,
- the lower quartile is 0.8 kg,
- the upper quartile is 2.4 kg.

This information can now be represented as a box plot.

Begin by drawing a horizontal line and marking a scale from 0 to 6 kg.
Above your line, draw a box from the lower quartile to the upper quartile and mark in the median with a line across the box.
Draw lines (sometimes called whiskers) from the lower end of the box to 0.2 kg and from the upper end of the box to 5.6 kg to represent the range.

A box plot for the masses of these fish is shown below.
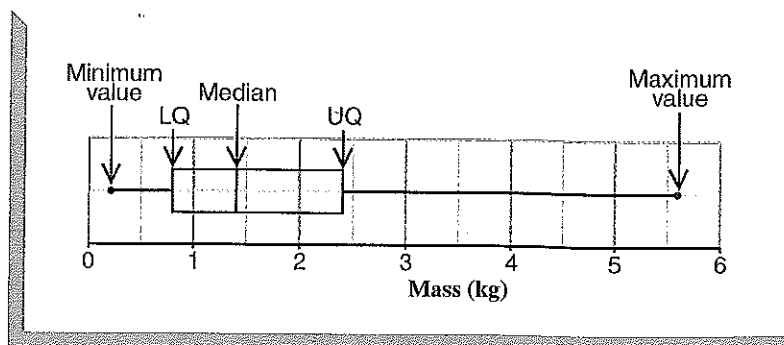
The box plot shows how the masses of these fish are spread out and how the middle 50% are clustered.

440

EXAMPLE

The number of points scored by the 11 players in a basketball team during a competition are shown.

13   12   5   34   8   10   11   46   25 -   23   4

Draw a box plot to illustrate the data.

Begin by putting the data in order and then locate the median, lower quartile and upper quartile.

4   5   8   10   11   12   13   23   25   34   46

↑ LQ   ↑ Median   ↑ UQ

Then use these values to draw the box plot.

0   10   20   30   40   50
Points

Exercise 39.4

① The midday temperatures, in °C, for 11 cities around the world are:

9   12   20   24   25   28   28   30   31   32   35

Draw a box plot to represent these temperatures.

② A group of 15 people were asked to estimate the weight of a large tortoise. Their estimates, in kilograms, are shown.

2.8   3.0   3.2   3.3   3.5   3.6   3.8   3.8
4.0   4.0   4.2   4.2   4.4   4.5   4.7

Draw a box plot to represent these estimates.

③ The cumulative frequency graph shows information about the distances travelled by children to get to school.

Draw a box plot to illustrate the data.

4 A sample of 23 people were asked to record the amount they spent on food last week. The amounts, in £'s, are shown.

37	46	55	63	19	63	16	22	42	47	23	18
51	38	27	33	42	64	56	48	57	37	22	

(a) Find the median and quartiles of this distribution.
(b) Draw a box plot to represent the data.

5 The box plot illustrates the reaction times of a group of people.

0   10   20   30   40
Time (hundredths of a second)
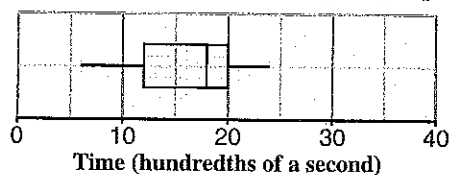
(a) What was the minimum reaction time?
(b) What is the value of the interquartile range?

6 A group of students took examinations in Mathematics and English.
The box plots illustrate the results.

Mathematics

English

10   20   30   40   50   60   70   80
Marks (%)
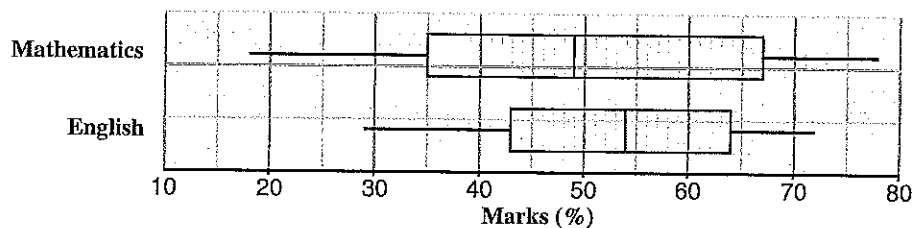
(a) What was the highest mark scored in English?
(b) What was the lowest mark scored in Mathematics?
(c) Which subject has the higher median mark?
(d) What is the value of the interquartile range for English?
(e) Comment on the results of these examinations.

7 The cumulative frequency graphs show the times taken by students to run 100 m.
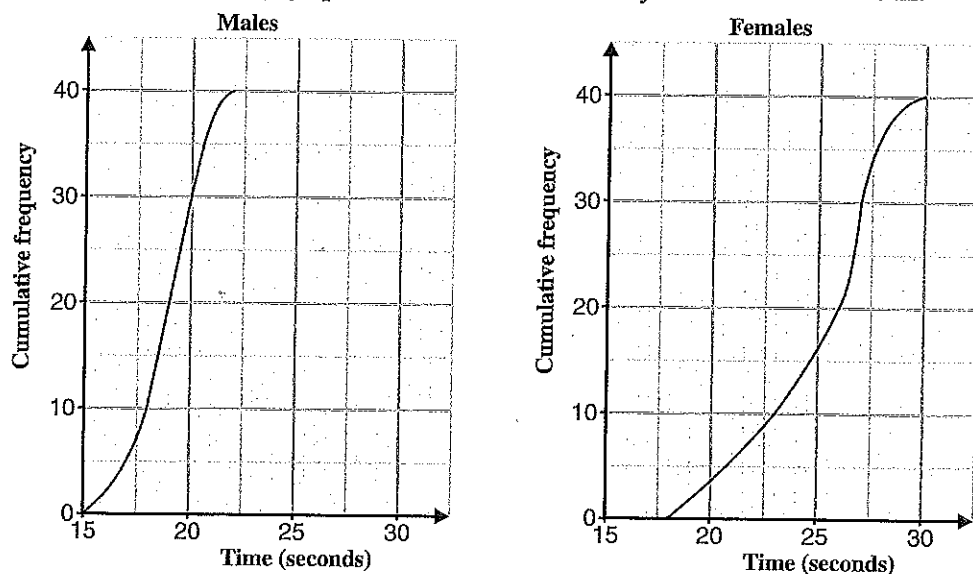
Males

Females

(a) Draw box plots to compare the times for males and females.
(b) Comment on the times for males and females.

ANSWERS

14. (a) E.g. To multiply by 200, multiply by 2 and then by 100.
    (b) E.g. To multiply by 2000, multiply by 2 then by 1000.

Exercise 1.5 — Page 7

1. (a) 16   (b) 20   (c) 8
2. (a) 17   (b) 157   (c) 136
   (d) 75 remainder 5   (e) 393 remainder 2
   (f) 206   (g) 1098   (h) 20 140
3. (a) 456   (b) 465   (c) 64
   (d) 654
4. (a) 100   (b) 702 000   (c) 10
5. (a) 8   (b) 15   (c) 12
   (d) 5   (e) 7
6. (a) 253   (b) 79   (c) 537
   (d) 126   (e) 45   (f) 613
7. (a) E.g. To divide by 200, divide by 100 then by 2.
   (b) (i) 26   (ii) 293   (iii) 412

Exercise 1.6 — Page 8

1. 1344
2. 2432
3. 4862
4. 11 130
5. 38 772
6. 76 812
7. 345 015
8. 6 620 544
9. 463
10. 158
11. 123
12. 256
13. 504
14. 654
15. 406
16. 208
17. £1392
18. (a) £1764   (b) £3427   (c) £9866
19. 1416
20. (a) 16 rem 10   (b) 25 rem 7   (c) 17 rem 35
21. 17, 7p change
22. (a) 41   (b) 16

Exercise 1.7 — Page 9

1. (a) 37   (b) 3   (c) 9   (d) 58
   (e) 6   (f) 30   (g) 5   (h) 19
   (i) 14   (j) 20   (k) 24   (l) 4
   (m) 0   (n) 56   (o) 56   (p) 6
   (q) 28   (r) 0
2. (a) $5 \times 6 + 7 = 37$
   (b) $5 + 6 \times 7 = 47$
   (c) $15 + 8 \times 9 = 87$
   (d) $15 \times 8 + 9 = 129$
   (e) $15 \times 8 - 9 = 111$
   (f) $15 \div 5 + 3 = 6$
   (g) $5 - 24 \div 6 = 1$
   (h) $19 \div 19 + 7 \times 0 = 1$
   (i) $4 \times 4 + 7 \times 2 = 30$

3. Many answers, for example:
   $6 - 3 \times 2 + 1 = 1$ $6 - 3 - 2 + 1 = 2$
   $6 \div 3 + 2 - 1 = 3$ $6 \div 3 + 2 \times 1 = 4$
   $6 - 3 + 2 \times 1 = 5$ $6 + 3 - 2 - 1 = 6$
   $6 + 3 - 2 \times 1 = 7$ $6 \times 3 \div 2 - 1 = 8$
   $6 \times 3 \div 2 \times 1 = 9$ $6 + 3 + 2 - 1 = 10$
4. 25   5. 148 cm   6. (a) 63   (b) 75

Review Exercise 1 — Page 10

1. (a) eight hundred and seventy thousand three hundred and two
   (b) 3 027 409
2. (a) 987 542   (b) 245 789
3. (a) 97, 404   (b) 114, 306
   (c) 92, 209
4. (a) 2059   (b) 587
5. (a) 73 500   (b) 6420
   (c) 3020   (d) 462
6. 298 miles
7. (a) 6462   (b) 241
8. (a) 50   (b) 50   (c) 10   (d) 2
9. £547.50
10. 14 664
11. 120
12. (a) 166   (b) 4
13. £14 522
14. £3427
15. £17
16. 32
17. 50

CHAPTER 2

Exercise 2.1 — Page 13

1. (a) $7 + 0.6 + 0.02$
   (b) $30 + 7 + 0.9 + 0.02 + 0.008$
   (c) $7 + 0.5 + 0.04 + 0.001$
   (d) $20 + 0.5 + 0.003$
2. (a) 0.4   (b) 0.009
   (c) 80   (d) 0.05
3. (a) 1.68   (b) 1.39   (c) 1.04
4. (a) 0.07   (b) 0.6
5. (a) A 10.5, B 11   (b) C 5.2, D 5.6
   (c) E 0.54, F 0.59   (d) G 0.751, H 0.757
6. (a) 3.001, 3.01, 3.1, 3.15, 3.2
   (b) 3.567, 3.576, 3.657, 3.675
   (c) 0.1, 0.15, 0.45, 0.5, 0.55
7. (a) 9.87, 9.78, 8.97, 8.79
   (b) 1.5, 0.15, 0.015, 0.00015
   (c) 2.701, 2.7, 2.67, 2.599
8. 47.5074   9. 93.07

12. (a) 115 (b) 175 (c) 45

(d)

n	$n+1$	$n+2$

$n+11$

$n+21$

(e) $S_n = 5n + 35$

Exercise 14.2

1. (a) 5 (b) 10 (c) 1
2. (a) 20 (b) 28 (c) −12
3. (a) 1 (b) 2 (c) −6
4. (a) 2 (b) −2 (c) 8
5. (a) 13 (b) −7
6. (a) −2 (b) 10
7. (a) 2 (b) 1 (c) $\frac{1}{2}$
8. (a) 1 (b) 1.5 (c) 2.5
9. (a) 6 (b) 5.8
10. (a) 0 (b) 1
11. (a) 3.5 (b) 4
12. $F = 75$ **13.** $V = 26$ **14.** $P = -9$
15. (a) $C = 104$ (b) $C = 32$
16. (a) $S = 40$ (b) $S = -2$ (c) $S = 6$
17. (a) $T = 35$ (b) $T = -2$
(c) $T = -18$ (d) $T = 6$
18. (a) $K = -1$ (b) $K = 13$
19. (a) $L = 10$ (b) $L = -11$
20. 33
21. (a) 96 m (b) 720 m (c) 36 m
22. (a) −10°C (b) −20°C (c) −30°C
(d) −40°C
23. (a) 50°F (b) 14°F (c) −22°F
(d) −40°F
24. 138 minutes
25. 240 volts **27.** £40.14
26. $F = 4100$ **28.** 0.3

Exercise 14.3

1. (a) $S = 9$ (b) $S = 9$
2. (a) $S = 18$ (b) $S = 18$
3. (a) $S = 36$ (b) $S = 36$
4. (a) $S = 32$ (b) $S = 32$
5. (a) $S = 16$ (b) $S = 16$
6. (a) 39 (b) 66
(c) 39 (d) 66

496

7. (a) 27 (b) −27
(c) 125 (d) −125
8. (a) 128 (b) −128
9. (a) −2 (b) 2
10. (a) $\frac{3}{4}$ (b) $1\frac{1}{2}$
11. (a) 4 (b) 1.5 (c) −3
12. (a) $L = 10$ (b) $L = 0.5$
13. (a) 26.0 (b) 3.2
14. (a) 12 (b) 1.4
15. $F = 144$ **16.** $h = 307.5$
17. (a) $T = 1.80$ (b) $T = 2.5$
18. (a) $v = 6.3$ (b) $v = 7.4$

Exercise 14.4

1. (a) $m = a - 5$ (b) $m = a - x$
(c) $m = a + 2$ (d) $m = a + b$
(e) $m = 2 - a$ (f) $m = x - a$

2. (a) $x = \frac{y}{4}$ (b) $x = \frac{y}{a}$
(c) $x = -\frac{y}{a}$ (d) $x = 2y$
(e) $x = ay$ (f) $x = -ay$
(g) $x = \frac{2}{y}$ (h) $x = -\frac{n}{y}$

3. (a) $p = \frac{1}{2}y - 3$ (b) $p = \frac{t - q}{5}$
(c) $p = \frac{m + 2}{3}$ (d) $p = \frac{q + r}{4}$
(e) $p = \frac{3 - a}{2}$ (f) $p = \frac{g - d}{3}$
(g) $p = 2m - 6$ (h) $p = 3y - 3x$
(i) $p = 5t + 10$ (j) $p = 2h - 2g$
(k) $p = \frac{3}{s - q}$ (l) $p = \frac{a}{x - y}$

4. $K = \frac{P}{0.45}$ **6.** $n = \frac{C - 35}{24}$

5. $c = \frac{F - 32}{1.8}$ **7.** $a = \frac{2A}{h} - b$

8. (a) $c = \pm \sqrt{y}$ (b) $c = y^2$
(c) $c = \pm \sqrt{\frac{y}{d}}$ (d) $c = 9y^2$
(e) $c = \pm \sqrt{y - x}$ (f) $c = (y - x)^2$
(g) $c = \pm \sqrt{dy - dx}$ (h) $c = (ax + ay)^2$

(c) $0.3\dot{6}$
(f) $0.21\dot{6}$

$i - i$ 135
285714

(c) $0.5\dot{1}$
(f) $0.85714\dot{2}$
(i) $0.8\dot{3}$

(c) 0.43
(f) 0.38

0.67
0.78

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4   **2.** 15

5. $\frac{4}{15}$ **6.** $\frac{7}{10}$

$\frac{9}{20}$ (d) $\frac{1}{8}$ **8.** $\frac{1}{4}$

(c) $2\frac{1}{10}$ (d) $2\frac{19}{30}$

(g) $1\frac{5}{7}$ (h) $2\frac{1}{2}$

11. $\frac{1}{10}$ **12** £80

(c) 0.42   (d) 0.402

4. 1.14, $1\frac{1}{7}$, 1.41, 11.14

16 208 km

Page 80

25%, 30%, 40%, 50%, 90%

(c) 15%   (d) 80%
(g) $66\frac{2}{3}\%$ (h) $22\frac{2}{9}\%$
(c) $\frac{9}{50}$ (d) $\frac{13}{25}$
(g) $\frac{57}{200}$ (h) $\frac{29}{40}$
(c) 32%   (d) 12.5%
(g) 1.5%   (h) $33.\dot{3}\%$
(c) 0.47   (f) 1.5

0.15
0.875

$\frac{9}{20}$

$\frac{57}{200}$, $\frac{23}{80}$

85%

8. 80%

(c) 88%   (d) 80%

11. Team A

1. (a) 60%   (b) 16%   (c) 70%
   (d) 21%   (e) 21%
2. (a) 32%   (b) 12.5%   (c) 50%
   (d) 90%   (e) 30%
3. 32%   **6.** (a) 40%   (b) 60%
4. 20%   **7.** 12.5%
5. 30%   **8.** 37.5%   **9.** 6%
10. (a) $33\frac{1}{3}\%$ (b) $66\frac{2}{3}\%$ (c) $11\frac{1}{9}\%$
   (d) $42\frac{1}{2}\%$ (e) $56\frac{1}{2}\%$
11. 9.5%   **12.** 25%

1. (a) £16   (b) £15   (c) £66
   (d) £52.50   (e) £25   (f) £30
   (g) £27   (h) 4 m   (i) 24 kg
   (j) 280   (k) £11.25   (l) 12
2. (a) 60   (b) 105   (c) 45%
3. £20   **4.** 270
5. (a) £42   (b) £70.50
6. £1.95   **7.** £4.50
8. (a) 660   (b) 198   **9.** 24 g
10. (a) £480   (b) £420   (c) £2800
   (d) £1080   (e) £3450   (f) £1260
   (g) £80   (h) £13   (i) £16.50
   (j) £57.50
11. (a) £420   (b) £600   (c) £2000
   (d) £150   (e) £10 200   (f) £4550
   (g) £510   (h) £5.50   (i) £33.60
   (j) £40.95
12. 24p per minute   **15.** 759 g
13. £215   **16.** £11 625
14. £18.45   **17.** 8400 cm²

1. £19.38
2. (a) £14 541.75   (b) 83.9p per litre
   (c) 32p per pint
3. (a) £244.64   (b) £4706   (c) £2372.50
4. $83\frac{1}{3}\%$ **5.** (a) £10 530   (b) 17.3%
6. £91 814   **7.** (a) £34.13   (b) £41.39
8. (a) 36.3%   (b) 23.7%   (c) 37.8%
9. 1.35×10^9 km³   **11.** 30%
10. 1.49×10^9 square miles   **12.** 2.53%

2. 5.3 kg

3. £94 000

4. 4.3 m

5. 27.7

1. Moneymaker larger range and higher mean than Cherry.

2. (a) (i) 0.5 minutes   (ii) 1.95 minutes
  (b) Girls a little slower on average and more varied.

3. Jays: mean 1.9, range 5
  Wasps: mean 2.4, range 3
  Wasps scored more on average and had less spread.

4. Women: mean 1.6, range 6
  Men: mean 1.5, range 2
  Women made more visits to the cinema, though the number of visits is more spread.

5. Average: Boys 6.2, Girls 7.2
  Range: Boys 4, Girls $4\frac{1}{2}$
  No. Girls' average greater than boys'.
  Correct about variation.

6. (a) MacQuick 20 - 29, Pizza Pit 30 - 39
  (b) MacQuick - mean 26 years
    (Pizza Pit 36.5 years)
  (c) Exact ages not known.

7. Before: median 3, range 4
  After: median 3, range 5
  Would have been better to calculate the means. Before 2.2, After 3.0

1. Mode trainers. Cannot calculate others.

2. Mode 15s, median 12s, mean 22.75s
  Median most sensible, not affected by 200 as is mean, mode not much use.

3. Mode 81, median 83, mean 69.8
  Median most sensible, not affected by 5 & 6 as is mean, mode not much use.

4. Swimmer A.
  Mean is lower (A 30.88s, B 31.38s)
  Range less (A 1.7s, B 15s)
  Median is higher (A 30.9s, B 30.0s)

5. Batsman B
  Higher median (B 31.5, A 21)
  Higher mean (B 36, A 35)

1. (a) (i) 9 minutes   (ii) 9.5 minutes
  (b) Trains are more variable, but less late on average.

2. (a) Range 6p, mode 31p
  (b) Median 30p, mean 30.1p

3. (a) (i) 2   (ii) 2.28
  (b) (i) 2.5   (ii) 2.63
  (c) The calculation for the mean involves the number of runs scored off every ball.

4. (a) 52   (b) 11   (c) Increase. $62 \div 9 = 6.9$

5. (a) 24 cm   (b) 24.3 cm
  (c) 1st group had higher mode (1st 25, 2nd 23), but lower range (1st 7, 2nd 13).

6. (a) £200 - £300   (b) £300 - £400
  (c) Mean, influenced by 12 people earning £600 - £1000.

7. 16.9 minutes

8. (a) $150 \leqslant p < 155$ (b) 156 litres
  (c) Estimated mean – uses all values

1. (a) 15°C
  (b) Temperature variations during each day are not known.
    Line only indicates trend in midday temperatures.

2. (a) 8   (b) Jan, Feb, June
  (c) No information given about when cars are sold during the month.

3. (b) (i) 154 cm   (ii) 15 years 4 months
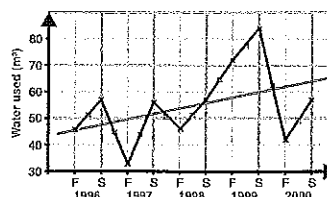
4. (a) 82 kg (b) 82 kg (c) 5 kg (d) Week 4

1. 35, 34, 35, 36, 39, 37, 40

2. 28.8, 28.6, 28.2, 28.6, 30.6, 33

3. 18, 17.5, 17, 16, 15, 14, 13, 12, 11.25   **4.** 6.6

5. 3-point moving averages are:
 370, 365, 375, 390, 400, 390, 400

1. (a) (c)

  (b) 51.5, 45, 44, 50.5, 51.5, 64.5, 77.5, 62.5, 49.5
  (d) There is an upward trend in the amount of water used.

4. (b) 9 to 10 hours   (c) 12%

5. (a) 175 cm, 14.5 cm   (b) 3   (c) 41
     (d) 172.5 cm   (e) 185.5 cm

Exercise 39.3   Page 439

1. (a) 40   (b) 13 cm   (c) 8.5 cm
     (d) Girls heights are more varied, etc.   (e) 1

2. (b) (i) 18 litres   (ii) 10 litres
     (c) Neighbour's cows have higher average yield, but the yield is more varied, etc.

3. Variety X, median 350 g, IQR 100 g.
Variety Y, median 350 g, IQR 220 g.
Same median, but weight of Variety Y is more varied, etc.

Exercise 39.4   Page 441

1.

Temperature (°C)

2.

Weight (kg)

3.

Distance (km)
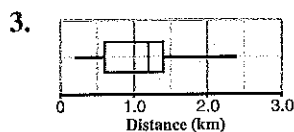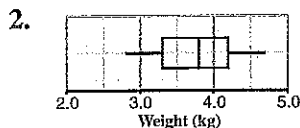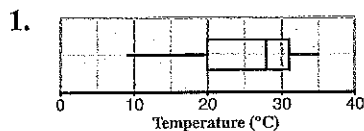
4. (a) median = £42, lower quartile = £23, upper quartile = £55

5. (a) 0.06 s   (b) 0.08 s

6. (a) 72%   (b) 18%   (c) English   (d) 21%
     (e) English mark has a higher median, Maths marks more spread.

7. (b) Males have lower median time and not spread out. Females have higher median time and greater variation.

Review Exercise 39   Page 444

1. (a) 3 miles   (b) 0.5 miles
     (c) Range is affected by a few long journeys, etc.

2. (b) (i) 57 marks   (ii) 22 marks
     (c) Paper 2. Higher median, less variation.

3. (a) Entries are: 2, 5, 13, 22, 28, 35, 40
     (c) 78.5 g   (d) 19 g
     (e) The spread of the second sample is greater (IQR 38 g compared with 19 g), etc.

4. (a) Lettuces planted by hand have a greater mean diameter.
     (b) Lettuces planted by machine have a smaller diameter and are less varied in size.
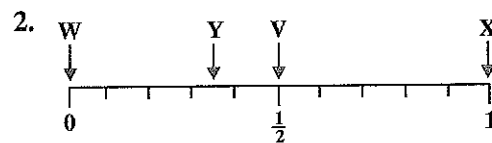
5. (a) 13p   (b) 27p   (c) 5p

6.

Time (minutes)
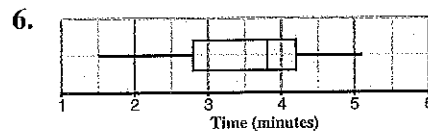
CHAPTER 40

Exercise 40.1   Page 446

1. (a) Evens   (b) Impossible
     (c) Likely   (d) Unlikely
     (e) Certain   (f) Unlikely
     (g) Likely

2.

Exercise 40.2   Page 448

1. (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{2}{3}$

2. (a) $\frac{3}{10}$ (b) $\frac{7}{10}$

3. (a) $\frac{1}{5}$ (b) $\frac{2}{5}$

4. (a) $\frac{1}{11}$ (b) $\frac{4}{11}$ (c) $\frac{2}{11}$

5. (a) $\frac{1}{12}$ (b) $\frac{1}{6}$

6. (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{52}$

7. (a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) 1   (d) 0

8. (a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$

9. (a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{25}$ (d) $\frac{4}{15}$
     (e) $\frac{2}{5}$

10. (a) $\frac{1}{3}$ (b) $\frac{1}{15}$ (c) $\frac{11}{24}$ (d) $\frac{17}{50}$
     (e) $\frac{21}{25}$ (f) $\frac{4}{5}$

11. The events are not equally likely.

Exercise 40.3   Page 450

1. $\frac{7}{25}$ **2.** $\frac{4}{5}$ **3.** $\frac{21}{30} = \frac{7}{10}$

4. (a) $\frac{1}{10}$ (b) 10